THE KINETICS OF SOMIGLIANI DISLOCATION

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1. To describe the structure and different plastic molding processes, as well as the fracture of real solids, there occurs the necessity to introduce a more general than usual (translational) dislocation of the type of the Somigliani dislocation [1] into the considerations. Questions of constructing the kinetics of such defects are indeed the subject of investigations in this paper. Some of these questions have already been examined earlier [2]. Here an approach independent of [2] is developed, in which Somigliani dislocations are introduced as a natural extension of the ordinary dislocations. The advantage of this approach is the possibility of using the well-developed apparatus of continual theory of ordinary defects (dislocations and disclinations). By using the results of [3, 4], an expression is successfully written here for the dynamic elastic fields of Somigliani dislocations and a closed system of kinetic equations is obtained.

2. A Somigliani dislocation is an extension of the Volterra dislocation [1, 5] and is usually determined [2] as a surface S on which the total displacement in an elastic body u_i^T undergoes a jump $[u_i^T]$ that changes arbitrarily along S

$$[u_l^T] = -B_l,$$

where B_l is the Burgers vector of Somigliani dislocation. The jump in the displacement is defined as the jump in the displacement upon going through the surface S in the direction of the normal to the surface n_k. Here the surface S can depend on the time t (S = S(t)); however, for simplicity in the writing, we will sometimes omit the symbol for the argument t.

Another possibility also exists for determining the Somigliani dislocation, namely, in terms of the basis plastic fields as is done in the continual theory of Volterra dislocations [3, 5]. In the case of ordinary Volterra translational dislocations, it is necessary to assign basis plastic distortion β_{kl}^P and velocity v_l^P fields of the form

$$\beta_{kl}^P(\mathbf{r}, t) = -\int_S \delta(\mathbf{R})\, b_l dS_k; \tag{2.1}$$

$$v_l^P(\mathbf{r}, t) = \int_S \delta(\mathbf{R})\, b_l v_k(\mathbf{r}', t)\, dS_k, \tag{2.2}$$

where $\delta(\mathbf{R})$ is the three-dimensional Dirac delta function ($\mathbf{R} = \mathbf{r} - \mathbf{r}'$), $\mathbf{r}$, $\mathbf{r}'$ are the radius vectors of the points of observation and integration, $v_k(\mathbf{r}, t)$ is the velocity of surface S(t) motion, and b_l is the constant Burgers vector of the dislocations. We shall consider the Somigliani dislocation to be an extension of the translational Volterra dislocations, and we represent the basis fields by expressions of the type (2.1) and (2.2) in which instead of the constant Burgers vector b_l there will now be $B_l(\mathbf{r}, t)$, the Burgers vector which changes along S(t)

$$\beta_{kl}^P(\mathbf{r}, t) = -\int_S \delta(\mathbf{R})\, B_l(\mathbf{r}', t)\, dS_k; \tag{2.3}$$

$$v_l^P(\mathbf{r}, t) = \int_S \delta(\mathbf{R})\, B_l(\mathbf{r}', t)\, v_k(\mathbf{r}', t)\, dS_k. \tag{2.4}$$

In order to set off the accepted viewpoint better, we examine the particular case of Somigliani dislocations when the edges of the surface S(t) are rigidly rotated a certain angle Ω_q with respect to each other, where

$$[u_l^T] = -B_l = -\varepsilon_{lqr}\Omega_q(x_r - x_r^0). \tag{2.5}$$

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Here ε_{lqr} is the unit antisymmetric tensor, x_r, x_r^0 are Cartesian coordinates of the point of observation and a point of the axis of rotation. Such a nature of the displacement jump corresponds to disclination [5]. In continual theory, the basis plastic disclination fields are given by four quantities [3]: the plastic strain tensor e_{kl}^P, the plastic bending-twisting tensor $\varkappa_{pq}^P$, the velocity vector of the plastic displacement v_l^P, and the rotation w_q^P. This is related to the fact that the plastic distortion tensor β_{kl}^P is considered unknown in the case of disclination. Opposing this we here consider (2.3) known for a Somigliani dislocation with a displacement jump (2.5). Therefore, only the so-called dislocation model of disclination [5] results as a particular case from the theory of Somigliani dislocations.

If the sense of the plastic distortions (2.3) is clear, then the sense of the plastic velocities (2.4) requires disclosure. Let us consider the plastic displacement field $u_l^P(\mathbf{r}, t)$ of the singular form

$$u_l^P(\mathbf{r}, t) = \int_V \delta(\mathbf{R}) B_l(\mathbf{r}')\, dV, \tag{2.6}$$

where V is the volume bounded by a closed moving surface S(t) while the vector B_l is independent of the time. As before, we determine that the dislocation density tensor α_{pl} defined by the relationship [5]

$$\alpha_{pl} = -\varepsilon_{pmk}\beta_{kl,m}^P, \tag{2.7}$$

equals zero (the subscript after the comma denotes differentiation with respect to the corresponding Cartesian coordinate). Indeed, we obtain from (2.6) for the plastic distorsion tensor β_{kl}^P

$$\beta_{kl}^P = u_{l,k}^P = \int_V B_l \delta_{,k}(\mathbf{R})\, dV. \tag{2.8}$$

Substituting (2.8) into (2.7), we find

$$\alpha_{pl} = -\varepsilon_{pmk}\int_V B_l \delta_{,km}(\mathbf{R})\, dV = 0. \tag{2.9}$$

Furthermore, we have for the velocity of the plastic displacements v_l^P (the dot above denotes differentiation with respect to time)

$$v_l^P = \dot{u}_l^P = \int_S B_l \delta(\mathbf{R})\, v_R dS_k. \tag{2.10}$$

As is seen from (2.10), the velocity of plastic displacement differs from zero for the field (2.6) only on the surface S(t) and has the form (2.4). Utilizing the ordinary relationship for the dislocation flux density J_{kl} [6] (this expression differs by the sign in [6])

$$J_{kl} = \dot{\beta}_{kl}^P, \tag{2.11}$$

we obtain (the subscript with the prime after the comma denotes differentiation with respect to the variable of integration)

$$J_{kl} = -\int_S B_l \delta_{,k'}(\mathbf{R})\, v_i dS_i. \tag{2.12}$$

As is seen from (2.12), the dislocation flux density tensor J_{kl} for the field (2.6) differs from zero and is concentrated on the surface S(t). Since $\alpha_{pl} = 0$, there are no elastic fields in statics. In dynamics, although $J_{kl} \neq 0$ it can also be assumed (postulated) that there are no elastic fields (see [3]). Therefore, the plastic displacement velocity v_l^P (2.4) can be interpreted as the velocity of plastic displacement associated with a plastic field of displacements of the form (2.6) in a certain body volume adjacent to S(t). As in [3], we here postulate that the plastic velocity field does not evoke elastic stress fields. Giving the Somigliani dislocations by two basis fields (2.3) and (2.4) means in this connection that an additional distortion of the medium whose velocity is $v_{l,k}^P$ is imposed on the plastic distortion of the defect (2.3).

Now the fundamental kinematic relationships can be written. The complete dislocation density tensor α_{pl} for the Somigliani dislocation (more accurately, for a medium with a Somigliani dislocation) will be comprised of components related to the distortions of the defect (2.3) itself and of the medium. As has been shown above in (2.9), the latter component equals zero; consequently, α_{pl} is determined by the relationship (2.7) in which β^P_{kl} is the plastic distortion of the defect (2.3). Substituting (2.3) into (2.7), we obtain for the dislocation density tensor α_{pl} of the Somigliani dislocation

$$\alpha_{pl} = \int_L \tau_p B_l \delta(\mathbf{R})\, dL + \int_S \varepsilon_{pmk} B_{l,m'} \delta(\mathbf{R})\, dS_k, \tag{2.13}$$

where L(t) is a closed contour bounding S(t), and τ_p is the unit vector tangent to the contour L whose direction is in agreement with the direction of the normal to the surface S. Here and henceforth, $B_{l,m}$ the gradient of the field B_l is in the formulas. Since the field B_l is given only on the surface S(t), a certain explanation of this concept is required. Points on the surface S(t) can be made individual by two parameters which we denote by ξ, η. We then have at each point M of the surface S

$$B_l = B_l(\xi, \eta, t); \tag{2.14}$$

$$\mathbf{r}_M = \mathbf{r}_M(\xi, \eta, t), \tag{2.15}$$

where r_M is the radius-vector of the point M. We shall assume that the dependence (2.15) is mutually one-to-one, at least for sufficiently small time intervals Δt (the case of a fixed surface requires special consideration). Then eliminating ξ, η, t from (2.14) by using (2.15), we obtain

$$B_l = B_l(\mathbf{r}_M). \tag{2.16}$$

The field (2.16) is a function of the space coordinates, and we will understand $B_{l,m}$ to be the gradient of the field (2.15). If it is convenient to use the parametric notation (2.14) for the field B_l for some reason, then the gradient B_l will be determined by the expression (in line notations)

$$\nabla \mathbf{B} = \nabla_S \mathbf{B} + \frac{\mathbf{n}}{(\mathbf{n}\cdot\mathbf{v})}\left(\frac{\partial \mathbf{B}}{\partial t} - \mathbf{v}\cdot\nabla_S \mathbf{B}\right), \tag{2.17}$$

where ∇, ∇_S are the spatial and surface nabla operators.

The complete dislocation flux density tensor J_{kl} will also consist of two components. According to (2.11) the first is determined by the plastic distorsion velocity of the defect $\dot\beta^P_{kl}$. The second occurs from the imposition of the field (2.4) with reverse sign and equal to $v^P_{l,k}$. We hence have for the dislocation flux density tensor

$$J_{kl} = \dot\beta^P_{kl} - v^P_{l,k}. \tag{2.18}$$

Substituting (2.3) and (2.4) into (2.18), we find

$$J_{kl} = -\int_S B_{l,k'} \delta(\mathbf{R})\, v_j dS_j + \int_L \delta(\mathbf{R})\, B_l \varepsilon_{pmk} \tau_p v_m dL. \tag{2.19}$$

Formulas (2.13)-(2.19) are valid only in the case when the velocity of Somigliani dislocation motion v_i does not vanish anywhere on S. In particular, as is seen from (2.17), $\nabla\mathbf{B}$ has a singularity as $\mathbf{v} \to 0$. However, if we substitute (2.17) into (2.13) and (2.19), then the singularity vanishes and formulas are obtained that are even valid for $\mathbf{v} = 0$:

$$\alpha_{pl} = \int_L \tau_p B_l \delta(\mathbf{R})\, dL + \int_S \varepsilon_{pmk} (\nabla_S \mathbf{B})_{lm} \delta(\mathbf{R})\, dS_k; \tag{2.20}$$

$$J_{kl} = \int_L \delta(\mathbf{R})\, B_l \varepsilon_{pmk} \tau_p v_m dL - \int_S (\nabla_S \mathbf{B})_{kl} \delta(\mathbf{R})\, v_j dS_j - \int_S \left[\frac{\partial \mathbf{B}}{\partial t} - v_m (\nabla_S \mathbf{B})_{ml}\right] \delta(\mathbf{R})\, dS_k, \tag{2.21}$$

where $(\nabla_S \mathbf{B})_{ml}$ are the surface gradient components. Furthermore, an expression for the plastic strain rate tensor $\dot e^P_{kl} = J_{(kl)}$, the plastic rotation rate tensor $\dot w^P_{kl} = J_{[kl]}$, and the rate of production of excess volume $\dot V = J_{kk}$ can be obtained. The fundamental kinematic relationships for the Somigliani dislocations are thereby exhausted. Formulas (2.20) and (2.21) express the characteristics of the continual theory of defects (dislocation density and flux) in terms of the field of displacement jumps B_l on the surface S(t).

3. To formulate the system of equations describing the time evolution of the Somigliani dislocations, it is furthermore necessary to write equations for the dynamic self-consistent

fields of elastic stresses, generalized forces, and velocities as well as the balance equations for the defect distribution functions.

As soon as the expressions for the dislocation densities and fluxes (2.20) and (2.21) are known, the elastic distortions can be calculated by the general formulas of continual theory [4]:

$$\beta_{mn}(\mathbf{r}, t) = \int [\varepsilon_{pmk} c_{ijkl} G_{jn,i}(\mathbf{R}, T)\, \alpha_{pl}(\mathbf{r}', t') - \rho \dot{G}_{ln}(\mathbf{R}, T)\, J_{ml}(\mathbf{r}', t')]\, dr'dt', \tag{3.1}$$

where $T = t - t'$; c_{ijkl} is the tensor of elastic moduli; G_{jn} is the dynamic Green's function; ρ is the mass density. The elastic stresses on the body σ_{ij}, induced by the Somigliani dislocation, are found by substituting the elastic distortions into Hooke's law:

$$\sigma_{ij} = c_{ijmn}\beta_{mn}. \tag{3.2}$$

Let us consider the question of the forces acting on a Somigliani dislocation. In the general case the motion of a Somigliani dislocation includes both a change in the configuration and position of the surface S(t) and a change in the field B_l. To be able to introduce the defect distribution function and write a balance equation for it, we are obliged to limit the number of allowable degrees of freedom of the Somigliani dislocations. This can be done by assuming that the configuration, location, and field B_l of the Somigliani dislocations in the ensemble under consideration are determined completely by the assignment of a finite number of parameters (generalized coordinates) q_k ($k = 1, 2, \ldots, N + 3$), where the last three generalized coordinates correspond to the Cartesian coordinates of the radius-vector $\mathbf{r}$ of the defect location. Such a method was utilized earlier in describing microcracks as a modification of a Somigliani dislocation in [7]. Moreover, we will consider only conservative motion not associated with the formation of excess volume since otherwise it is necessary to include point defects responsible for mass transfer.

The potential function $\Pi = \Pi(q_1, q_2, \ldots, q_{N+3})$ for the elastic forces has the form

$$\Pi = W - A,$$

where W is the intrinsic elastic energy of the defect, and A is the work of all the external (with respect to the defect under consideration) forces on the plastic displacements. The expression

$$dA = \int \sigma_{ij}^{+} \frac{\partial \beta_{ij}^{+P}}{\partial q_k} dq_k dV$$

can be written for the differential of the work dA, where σ_{ij}^{+} is the total stress in the body, and β_{ij}^{+P} is the total plastic distorsion of the body with the defect, and integration is over the volume included in the defect itself. We hence obtain for the generalized force F_k

$$F_k = -\frac{\partial \Pi}{\partial q_k} = -\frac{\partial W}{\partial q_k} + \int\limits_V \sigma_{ij}^{+} \frac{\partial \beta_{ij}^{+P}}{\partial q_k} dV. \tag{3.3}$$

The first term in the right side of (3.3) is the self-action force F_k^S, while the second is the external force F_k^e. It is clear from the method of introduction of the generalized coordinates that the vectors $\partial B_i/\partial t$ and v_m introduced in (2.21) are linear functions of the generalized velocities $\dot{q}_k = dq_k/dt$. Consequently $J_{ij} = J_{ijk}^{+}\dot{q}_k$ is also a linear function of $\dot{q}_k$ with coefficients J_{ijk}^{+}. By using the rule of differentiation with respect to the parameter t, the partial derivative $\partial\beta_{ij}^{+P}/\partial q_k$ can be replaced by the quantity $\partial J_{ij}/\partial \dot{q}_k = J_{ijk}^{+}$. Then the final expression for the generalized force becomes

$$F_k = F_k^S + \int\limits_V \sigma_{ij}^{+} J_{ijk}^{+} dV. \tag{3.4}$$

As an illustration, let us consider the motion of an element $\tau_p dL$ of translational dislocation which is described by three generalized coordinates x_k, the Cartesian coordinates of the location radius-vector of the dislocation. Taking into account that $\dot{x}_k = v_k$, $B_l = b_l$ = const we find the expression for the external force $F_m^e = \varepsilon_{pmk} b_l \sigma_{lk}^{+} \tau_p dL$ that agrees with the known Peach–Keler formula [6].

To describe the behavior of the ensemble of Somigliani dislocations, we introduce the distribution function $f(q_1, q_2 \ldots, q_N); \mathbf{r}, t)$ such that the number of Somigliani dislocations with generalized coordinates between q_k and $q_k + dq_k$ in the volume element $d\mathbf{r}$ is equal to $f(q_1, q_2, \ldots, q_N; \mathbf{r}, t) dq_1, \ldots, dq_n d\mathbf{r}$. We write the balance equation for f

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial q_k}(\dot{q}_k f) = I, \tag{3.5}$$

where $\dot{q}_k$ is the velocity of defect motion in (N + 3)-space, and I is the collision integral that takes account of the generation-annihilation process and other discrete defect transformations. It is assumed that the generalized velocities $\dot{q}_k$ are functions of the generalized forces

$$\dot{q}_k = \varphi_k(F_1, F_2, \ldots, F_{N+3}), \tag{3.6}$$

where $\dot{q}_k$ should be determined from microscopic theory or from experiment. Underlying the assumption (3.6) is that the defects move sufficiently slowly and homogeneously, and the forces of inertia are taken into account well by the mean dynamical stress fields σ^+_{ij} [8]. The generalized forces F_k of (3.4) are determined by the total stresses σ^+_{ij} which are comprised of the external applied σ^0_{ij} and the internal σ_{ij} stresses:

$$\sigma^+_{ij} = \sigma^0_{ij} + \sigma_{ij}. \tag{3.7}$$

For the self-consistency of the problem there remains to express the internal stresses σ_{ij} in terms of the Somigliani dislocation distribution function. To this end, by using an averaging procedure we introduce the mean plastic distorsion $\bar{\beta}^P_{kl}$ and velocity $\bar{v}^P_l$ fields:

$$\bar{\beta}^P_{kl} = \int \beta^{P1}_{kl}(q_1, \ldots, q_N) f(q_1, \ldots, q_N; \mathbf{r}, t)\, dq_1 \ldots dq_N; \tag{3.8}$$

$$\bar{v}^P_l = \int v^{P1}_l(q_1, \ldots, q_N) f(q_1, \ldots, q_N; \mathbf{r}, t)\, dq_1 \ldots dq_N, \tag{3.9}$$

where β^{P1}_{kl}, v^{P1}_l are the integral characteristics of the defects.

$$\beta^{P1}_{kl} = \int_S \beta^P_{kl} dS, \quad v^{P1}_l = \int_S v^P_l dS.$$

The elastic distorsions β_{mn}, caused by the plastic fields (3.8) and (3.9), are found from (3.1) in which α_{pl} and J_{ml} should be expressed in terms of $\bar{\beta}^P_{kl}$ and $\bar{v}^P_l$ by using the relationships (2.7) and (2.18). Then, by substituting the obtained values of β_{mn} in the Hooke's law (3.2), we find the desired stresses σ_{ij}.

Therefore, the balance equation (3.5), the motion laws (3.6), and the formulas (3.4), (3.7)-(3.9), (3.1), (2.7), and (2.18) to find the generalized forces comprise a closed system of equations describing the kinetics of an ensemble of Somigliani dislocations.

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